

SHUFFLED FROG LEAPING ALGORITHM FOR SOLVING OPTIMIZATION OF TOTAL HOLDING COST IN JOB SHOP SCHEDULING PROBLEMS

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ABSTRACT

The manufacturing System is enabled with an excellent knowledge of the production plan and proper scheduling of machinery process. Challenging combinatorial optimization problems are encountered even in the job shop scheduling problems. Heuristics algorithms are developed in a scholastic search way in which natural big buoyancy is maintained. These are developed to bring optimized results in stipulated time with respect to optimally schedule. This article deals with minimizing the total holding cost of completed and in-process products with consideration of tardy jobs and without consideration of tardy jobs with Shuffled Frog Leaping Algorithm (SFL). Applying SFL algorithm to minimize the total holding cost which is the sum of earliness finished product inventory holding cost, intermediate inventory holding costs, work in process and tardiness. Several benchmark problems of different sizes, which are commonly used for Job Shop Scheduling Problems of minimizing the total holding costs and makespan are produced. The Results are compared with literature results in terms of total holding cost, stipulated time and computational time the Shuffled Frog Leaping algorithm performs result oriented than other Heuristics Algorithm.

KEYWORDS: *Heuristic Algorithm, Job Shop Scheduling, Shuffled Frog Leaping Algorithm, Tardy Jobs & Total Holding Cost*

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I. INTRODUCTION

A schedule is an allocation of tasks to the time intervals on the machines. The aim is to find a schedule that minimizes the overall completion time, which is called the makespan. In the job shop scheduling, problem n jobs have to be processed on m different machines. Each job consists of a sequence of tasks that have to be processed during an uninterrupted time period of a fixed length on a given machine. Due dates are treated as deadlines and require the job-shop scheduling to meet specific due dates in order to avoid delay penalties, including customer's bad impression, cost of lost future sales and rush shipping cost.

Efficient Methods are traditional approaches consider technological advances in both processes and equipment as the key to success and the right way to remain competitive. Many valid approaches and its advances are compared and shared between competitors in rapid form. Each and every approach have its own valid solution

exclusively [1].

Akers proposed a graphical approach for the $2 \times m$ problem [2]. Hefetz et al. have developed an efficient approach for then $X \times 2$ problem where all operations are of unit processing time [3]. Williamson et al. proved that the feasible schedule determining with a makespan can be done in polynomial time [4].

The Job shop scheduling problem has been described as NP which means Non deterministic Polynomial time. Lenstra et al [5-6] solved the 3×3 problem, then $X \times 2$ instance with no more than 3 operations per job and then $X \times 3$ problem with no more than 2 operations per job. Lenstra et al. proved that then $X \times 2$ instance in which operations last for no more than 2 units of processing time and then $X \times 3$ problem in which all operations are of unit processing time belongs to the set of NP instances [7]. Mattfeld et al proposed randomly generated solutions with precedence relations which are not uniformly distributed [8]. A 10×10 problem proposed by Fisher et al. was solved by Carlier et al. [9]. Shmoys et al. proposed several poly-logarithmic approximations for evaluating an optimal schedule with makespan minimization criteria [10]. French predicted that no efficient algorithms will ever be developed for the majority of scheduling problems [11]. As a result, the focus of optimization research has turned to be enumerative approaches.

Branch and Bound (B&B) algorithms use a dynamically constructed tree structure as a means of representing the solution space of all feasible sequences. The branching procedure replaces an original problem by a set of new problems that are mutually exclusive and exhaustive subproblems, partially solved versions and smaller problems than the original. The two most common branching strategies were Generating Active Schedules (GAS) and Settling Essential Conflicts (SEC) [12]. Barcker et al. have generated Lower bounds by reducing job shop scheduling problem into subproblems of dimensionality [13]. The B&B search technique was initially studied by Brooks et al. [14]. Using a dynamically constructed tree structure represents the solution space of all feasible sequences Each node Search begins at the topmost level and a complete selection is achieved only at the lowest level has and (p) level in the search tree represents a sequence of operations partially. The branching operation sequences determine the next set of possible level nodes from sequence search progress [15].

Approximation procedures applied to a job shop problem were first developed on the basis of priority dispatching rules and due to their ease of implementation and substantially reduced computational requirement. Some of the Priority Dispatching Rules are Earliest Due Date (EDD), Shortest Processing Time (SPT), Minimum Slack, First (MSF), First come, first served (FCFS), Most work Remaining (MWR), Most Operations Remaining (MOR), Least Work Remaining (LWR), Random Analysis (RA). The earliest work on PDRs was done by Jackson [16]. The comprehensive survey of scheduling PDRs heuristics was done by Panwalker et al. where 113 PDRs were presented, reviewed and classified [17]. Haupt et al. provided an extended discussion and summary of these and many other PDRs [18]. The comparative study was given by Chang et al. who evaluated the performance of 42 PDRs using a linear programming model [19]. Lawrence compared the performance of ten individual priority dispatch rules with a randomized combination of these rules [20]. The results found from PDRs had more deviations from optimum. The results suggested that PDRs are more suitable as an initial solution technique rather than being considered as a complete JSP.

It has been recognized by many researchers that scheduling problems can be solved optimally using mathematical programming techniques and one of the most common forms of mathematical formulation for the job shop scheduling problem was the Mixed Integer Linear Programming (MIP) format of Manne [21]. Blazewicz et al. Emphasized the

difficulties of JSP and indicated that mathematical programming models have not been achieved enough breakthroughs for scheduling problems [22]. Any success that has been achieved using mathematical formulations can be attributed to Lagrangian Relaxation (LR) approaches and decomposition methods. Mathematical formulations were combined with other techniques and applied in the calculation of the lower bound. Results indicate that the lower bound generated was not optimal and it is difficult to calculate and takes excessive computing time. It is evident that mathematical approaches are inadequate for the job shop scheduling problem. Consequently, the main focus of enumerative approaches for the job-shop is a branch and bound techniques.

In earlier research, the job-shop scheduling problem has been extensively studied with the objective of minimizing some functions of the completion times of jobs. Several techniques have been proposed and different heuristics have been designed and developed for solving the minimum makespan problem, the minimum total tardiness problem and so on. SFHM algorithm was used for minimizing mean tardiness and mean flow time multiobjective criteria [23]. An effective SFLA was used for minimizing maximum completion time (i.e., makespan) [24]. In this work, SFLA and SFHM algorithm are used for solving the scheduling problem to meet due dates in a simple job shop. It is developed to approximately minimize the total holding cost which corresponds to the sum of product inventory cost and in-process inventory cost. Several benchmark problems are solved by the proposed algorithms and the results are compared with literature results.

II. JOB SHOP SCHEDULING

A. Scheduling Objectives

The scheduling is carried out to meet various objectives. These objectives are decided upon the situation, market demands, and the customer's satisfaction. The objectives considered under the time and cost minimization are listed in Table 1.

Table 1: Main Objectives of Scheduling

| Sl. No | Time Minimization | Cost Minimization |
|--------|---|---|
| 1 | Minimize machine idle time | Minimize the costs due to not meeting the due dates |
| 2 | Minimize the mean flow time | Minimize the maximum lateness of any job |
| 3 | Minimize the mean tardiness | Minimize the total holding cost with no tardy jobs |
| 4 | Finish each job as soon as possible | Minimize the total holding cost with tardy jobs |
| 5 | Finish the last job as soon as possible | Minimize the number of late jobs |

B. Job Shop Scheduling Problem

Typical scheduling problems involve minimizing the maximum $gj(t)$ value (the maximum cost problem) or minimizing the sum of $gj(t)$ values (the total cost problem). Scheduling is defined as the art of assigning resources to tasks in order to ensure the termination of these tasks in a reasonable amount of time (25). The term 'Scheduling' in

manufacturing systems is used for the determination of the sequence of operations in which parts are to be processed over the production stages. To meet an optimal objective solution or set of objectives these approaches are used for the determination of the starting time and finishing time of processing of each part. Some other cases scheduling, problem is addressed after the orders are released into the shop floor, along with their process plans and machine routings [26]. Scheduling plays a crucial role to increase the efficiency and productivity of the manufacturing system. The problem of scheduling is one of the operational issues to be addressed in the system on a daily or weekly basis. Job shop scheduling problems are Non-Polynomial (NP) hard] so it is difficult to find optimal solutions [27].

C. Single Machine Scheduling

In the single machine scheduling, problem n jobs have to be processed with a single operation. The concept of single machine scheduling consists certain conditions. At time zero a set of independent single operation jobs are available for processed, A set-up time of each job is independent, Job description are well known in advance, No machine kept idle when work is processing and each job is processed till its completion without a break. The basic data are necessary to describe jobs in a deterministic problem mentioned in the following Table.2.

Table 2: Basic Data's for Deterministic Problem

| Sl. No. | Basic Data & Representation | Description |
|---------|-----------------------------|---|
| 1 | Processing Time (t_j) | Time required to process job J . It will include both actual processing time and set-up time. |
| 2 | Ready Time (R_j) | Time at which job j is available for processing. It is the difference between arrival time and the time at which the job is processing. |
| 3 | Due Date (d_j) | Time at which the job j is to be completed. |
| 4 | Completion Time (C_j) | Time at which the job j is to be completed in sequence. |
| 5 | Flow Time (F_j) | Amount of time job j spends in the system ($F_j = C_j - r_j$) |
| 6 | Lateness (L_j) | Amount of time by which the completion time of job j differs from the due date ($L_j = C_j - d_j$) |
| 7 | Tardiness (t_j) | Lateness of job j if it fails to meet its due date, or zero $T_j = \max\{0, C_j - d_j\}$ |

D. Flow Shop Scheduling

In the Flow shop scheduling, problem n jobs, each job has to be processed on m different machines. The concept of single machine scheduling consists certain conditions. At time zero a set of multiple operation jobs are available for processing, A set-up times for the sequence operations are independent, Job description is well known in advance, m different machines are available for continuous processing and each individual operation of jobs are processed till its

completion without a break.

E. Job Shop Scheduling

In the job shop scheduling, problem n jobs have to be processed on m different machines. Each job consists of a sequence of tasks that have to be processed during an uninterrupted time period of a fixed length on a given machine. So the maximum of completion time needed for processing all jobs is subjected to the constraints that each job has a specified processing order through the machines and that each machine can process at most one job at a time.

F. NP-Hard Problems

Scheduling problem is NP-hard because it is little consolation for the algorithm designer who needs to solve the problem. By despite a theoretical evaluation all NP-hard problems are not equally hard from a practical evaluation. The NP-hard problems can be solved pseudo polynomial using dynamic programming. The sizes of the problems are not large enough to provide satisfactory results in heuristic approach.

G. Pseudo Polynomial Concept

Polynomial time concept exists for some NP-hard problems under the appropriate encoding of the problem data. Such problems are referred to as NP-hard in the ordinary sense and the algorithms are called pseudo polynomial Problem P is called strongly NP-hard if a pseudopolynomial concept for it does not exist. Algorithms which are Polynomial or Pseudo polynomial are applicable for single machine scheduling. Lawler (1973) developed Lawler's algorithm which constructs a sequence in a reverse to get an optimum sequence. The following criteria's are considered to get optimum solutions when the algorithms are pseudo polynomial [27]. Some existing criteria's are minimax criteria, Maximum lateness & related criteria, earliest due date criteria, Total weighted completion time criteria, Optimum sequence criteria, weighted number of late jobs and Total weighted tardiness.

III. MINIMIZATION OF TOTAL HOLDING COST

A. Total Holding Cost

Normally, manufacturing system consists active period starts from the first day of production on the machine with a certain set of actions and operations. In general Meeting, the due dates are the most important goal of scheduling to avoid the delay penalties, including customer's bad impression, lost future sales. Due-date oriented functions, whereas the main aim of optimizing the makespan is to minimize the total holding costs and maximizing the output.

During the production and process time the total holding cost is considered the raw materials holding cost, running process holding cost with stipulated time, labors cost based on the number of employees, machine cost may be owned machine or hired machine, power supply cost based on electricity utilization, Fuel cost, Transportation cost, stock holding cost and inventory management cost. Total accountability on every unit should be readily available in every set of job production. The total holding cost in every production should be equal to + or - 1 deviate from actual. When the production function starts the process management can be completed or to be completed with regular breakeven analysis by applying optimization methods to meet the regular benchmark of production cost management. Proper scheduling of machining processes and operations are enabled in master production schedule in which production to be reached on climax within the stipulated time with an excellent knowledge of production engineer. The employee to avoid absenteeism is to

bring out per capita per month analysis. One labor produce per day products worth of + or – k thousands. 16 x 3 Labors produces per day products worth 16 x 3 x 7 is equal to 336 k thousands (3.36 + or – L per day).

B. Minimization of Tardiness Cost

A tardy job is the sum of the rejected part cost and penalty cost. To minimize the total holding cost consider the minimization of work in the process holding cost with running holding cost, minimization of finished product inventory holding cost and minimization of the number of tardiness.

C. Heuristics Algorithms

The heuristic approaches were also employed for various engineering application problems due to their robustness and convergence to global optima. Heuristic method of learning involves discovery and problem solving using reasoning and past experience. An approach without formal guarantee of performance can be considered a “heuristic”. These heuristic approaches are used in a practical situation when no better methods are available. The following section deals with the various heuristic approaches like Artificial Intelligence, Bottleneck based heuristics, Local search approaches, Meta Heuristics and Hybrid Approaches in earlier research work.

Heuristics such as Tabu search, Hybrid shuffled frog leaping, Branch and Bound Technique, Bee colony optimization, Scatter search, Hybrid Metaheuristics, Shifting Bottleneck procedure, Shuffled Leaf Frog Algorithm, Ant colony optimization, and Greedy Randomized adaptive search. Such performance measures give us some insight into the reliability of a particular procedure.

D. Shuffled Frog Leaping Algorithm

Eusuff et al. [28] proposed a new meta-heuristic algorithm called Shuffled Frog Leaping Algorithm for solving scheduling problems with discrete decision variables. SFLA is a population-based cooperative search metaphor combining the benefits of the genetic-based memetic algorithm and the social behavior based particle swarm optimization Inspired by natural memetics [29]. Muzaffar Eusuff and Lansey [30] described the algorithm is imitating the total sequence of modeling process and searching for best food with behavior of frogs placed on separately positioned stones in a pond and also SFLA has been tested with a large number of combinatorial problems and found to be efficient in finding global solutions [31-32]. The SFLA is a population-based cooperative search metaphor inspired by natural memetics and consists of a frog leaping rule for local search and a memetic shuffling rule for global information exchange.

The SFLA comprises a set of an interacting virtual population of frogs partitioned into different groups population memplexes which are referred to searching for food [33]. The algorithm functions are simultaneously independent in local search of each memplex [34]. In terms of processing time and makespan, the SFLA compares the results rapid favorably with the Sheep Flock Heredity Model Algorithm, Artificial Intelligence System, Genetic Algorithm, and Particle Swarm Optimization [29, 35, and 36].

Mohammadreza Farahani et al. [29] identified a new hybrid algorithm called Hybrid Shuffled Leaping Frog Algorithm based on the identification of the weaknesses of the basic SFLA. At First, the SFLA is initially applied to different functions and to identify the fundamental weaknesses of this method as per the elimination of the effective frogs from memplexes by solving procedure in consequence order. This method is similar to the SFLA, partitions particles into different groups called memplexes and identified the best particle in each memplex thereafter determines its movement

through the search space in each iteration of the algorithm toward the global best particle and the worst particle in each memplex keeps track of its coordinates in the solution space by moving toward the local best particle in the same memplex.

Chen Fang et al.[37] proposed to encode concept for the virtual frog as the extended activity list and decode it by the SFLA-specific serial schedule generation scheme. The initial populations are identified and generated by the mutual based shuffling method and the priority rules. The large group of populated virtual frogs is separated into many sets of memplexes is the next stage and by applying the effective resource- based planning each memplex are evolves the crossover. Combining the permutation-based local search and forward-backward improvement is to enhance the improved exploitation ability. Virtual frogs are periodically shuffled and rearranged into new set off memplexes are maintained by the diversity of each memplex. HSLFA also has a distinct advantage over the SFLA in that it reduces the probability of the particles being trapped in the local minima by directing the best local particle toward the global best particle [34].

E. Drawbacks of Direct Approaches for Job Shop Scheduling

Job shop scheduling problems are NP-hard so that many direct approaches are not performing well due to more complexity of the problems. Many researchers are identified and observed valid drawbacks indirect approaches. Direct approaches are giving optimal solutions to very small size problems. In multi -objective optimization, direct approaches are not efficient. Indirect approaches, the convergence to an optimal solution depends on the chosen initial random solution and the results tend to stick with local optima. These techniques start with a single point and its follow a deterministic rule. Direct approaches are not efficient when practical search space is too large. Branch and Bound approach, Priority Dispatching Rules are giving solutions which have more deviation from optimal solutions. The above drawbacks stress the researchers to search, develop the efficient heuristic approaches.

F. Trends of Heuristic approaches for Job shop Scheduling

The heuristic approaches have more benefits compared with the direct approaches. The heuristic approaches produce the optimal solution for various size problems. Objective functions have given more importance than derivatives. Many Heuristic approaches use a population of points during the search. Initial populations are generated randomly which enable to explore the search space is large. These approaches efficiently explore the new combinations with the available knowledge to find a new generation.

Though an extreme work has been on solving JSS optimization using Metaheuristics and local search techniques are still the major potential area yet to be explored by the researchers using Efficient Heuristics Algorithms

IV. PROBLEM DESCRIPTION

A. Job Shop Scheduling Problem for Minimizing Total Holding Cost Subject to Consideration of Tardy Jobs

In-process cost is the sum of the running process cost, machining cost, raw material storage cost running cost, lateness, Earliness, Absolute deviation, squared deviation, unit penalty cost. Inventory holding cost is the sum of the work in the process holding and finished goods holding cost. A tardy job is the sum of the rejected part cost and penalty cost. To minimize the total holding cost consider the minimization of work in the process holding cost with running holding cost, minimization of finished product inventory holding cost and minimization of the number of tardiness.

Formulation of Objective Functions

Consider a non-preemptive job shop with m machines ($M_i = i, \dots, m$) & n jobs ($N_i = i, \dots, n$). When j_i is the set of a job to be processed on machine M_i . The operation sequence of the job j are denoted by O_{ij} , where i^{th} operation on j^{th} machines M_j . Objective functions depend on due date which is associated with the jobs. A job (j_i) consists of the number of (n_i) operations ($O_{i1}, O_{i2}, \dots, O_{in}$). The operation O_{ij} is a processing requirement P_{ij} and may be processed on any of machines in M_{ij} (set of machines) with ready time R_j . If each job, j_i consists only of one operation ($n_i = 1$) then the identity of job j_i with operation O_{i1} & the processing requirement P_i . In each job, k has a constant weight (cost). $w_k > 0$. In every problem of due date (d_k) consideration have almost different due dates which are NP complex, Where $k=1,2,\dots,n$. When $C_k > d_k$, the decision maker incur the penalty function of W_k . If the decision makers incurs no penalty for the k^{th} job is no tardy jobs (when C_k less than or equal to d_k). This case is formulated as multiple resource operations and schedule allocation problem with computation variables, which can be solved to optimally in polynomial and stipulated time with makespan. All the jobs ($j=1,2,\dots,n$) must be sequenced before processing of the jobs begins. The main objective is to optimize the number of job sequences with stipulated time S_k is $j_1 j_2, \dots, j_n$. All the jobs to be completed after the due dates end with makespan, which minimizes the expected weighted number of tardy jobs T_{ij} . The cost function c_{ij} , which measures the cost of completing j_i at the time (t), due date (d_i), Earliness cost (E_j) and defining function f_i .

Total holding Cost (h_j) = (in process cost + inventory cost) with tardy jobs cost

$$\min f = \sum_{k=1}^n \sum_{j=1}^m \{ \sum_{i=1}^n [t_{ij}^{(k)}, c_{ij}^{(k)}] x_{ij} + [t_{ij}^{(k)}, d_{ij}^{(k)}] x_{ij} + [t_{ij}^{(k)}, S_{ij}^{(k)}] x_{ij} + [R_{ij}] x_{ij} + [E_i, W_{ij}, c_{ij}] x_{ij} \} \quad \text{----- (1)}$$

Subject to

$$\begin{aligned} x_{ij} &= 1, 0 \\ d_{ij}^{(k)} &= i < k < p \\ I_{ij}^{(k)} &= 0 < k < l \\ t_{ij} &< 1 \text{ unit} \end{aligned}$$

For a given schedule (S), c_{ij} is the time at which job j finished processing on machine i and w_{ij} is the weighted time of job j spends in the queue before the first machine i . Already times, processing times and due dates are assumed to be integer. In the above function k^{th} job is performed in an i^{th} machine with j^{th} operation with unit time consideration for time t_i and cost j_{cost} . If the i^{th} machine is assigned with j^{th} operation for the first job is $X_{ij}^{(1)}$ is 1,0. If the i^{th} machine is assigned with j^{th} operation for the k^{th} job is $P_{ij}^{(k)}$ is 1,0. Sub-objective functions are also be formulated to minimize the tardiness cost t_j , Earliness cost E_j , finished product holding cost h_{ij} & w_{ij} and in-process holding cost P_{ij} . [38]

Consideration of Separate Objective Functions

Minimization of Tardiness cost function

$$C(s) = \sum_{i=1}^n \pi_i t_i \quad (2)$$

Minimization of Earliness cost

$$C(s) = \sum_{i=1}^n \epsilon_i E_i \quad (3)$$

Minimization of product holding cost

$$C(s) = \sum_{i=1}^n \sum_{j=2}^m h_{ij} w_i \quad (4)$$

Minimization of in process holding cost

$$C(s) = \sum_{j=1}^n h_{ij} (C_{ij} - P_{ij} - R_{ij}) \quad (5)$$

The first job cost is algebraic sum of Inventory holding cost ($I_{ij}^{(k)}$), Processing cost ($t_{ij}^{(k)}$).

For Job (1) = $I_{ij}^{(1)} + P_{ij}^{(1)} + t_{ij}^{(1)}$ where $I_{ij}^{(1)}$ is zero

For Job (2) = $I_{ij}^{(2)} + P_{ij}^{(2)} + t_{ij}^{(2)}$ where $I_{ij}^{(2)}$ is x

For Job (k) = $I_{ij}^{(k)} + P_{ij}^{(k)} + t_{ij}^{(k)}$

The sequence of jobs $J_{ij}^{(k)}$ have due dates to minimize the total holding cost with the number of tardy jobs to find the sequence of jobs. Further, For Solving the above objective functions to find an optimum solution Heuristics method named shuffled frog leaping algorithm has to be implemented and validated.

B. Job Shop Scheduling Problem for Minimizing Total Holding Cost Subject to no Tardy Jobs [39]

A set of I jobs has to be processed on K machines. The processing of job J_i ($i=1, 2, \dots, I$) on a machine is called an operation and each operation can be performed by only one machine. The processing order of a job is given. Let O^l_i ($l=1, 2, \dots, L_i$) denote the l^{th} operation of job J_i , where L_i corresponds to the number of operations for job J_i . The processing time p^l_i of operation O^l_i is pre-specified. Each machine k ($k=1, 2, \dots, K$) can process only one operation at a time. Pre-emption is not allowed, and each job is available for processing at time 0. The due date d_i of job J_i is pre-specified by the associated customer. Every job must be completed before or just on its due date and no tardy jobs are allowed. The holding cost in the shop floor incurs the in-process time once a job begins processing, and if a job is completed earlier than its due date, then earliness cost can be induced in the shop floor to holds the finished job. Assume $w^{l-1}_i < w^l_i$, where w^l_i ($l=1, 2, \dots, L_{i-1}$) denotes the holding cost per unit time for in-process product in idle time from end of operation O^l_i to start of operation O^{l+1}_i , and $w^{L_i}_i$ denotes the holding cost per unit time for completed product from end of operation $O^{L_i}_i$ to due date d_i . This assumption means that holding cost for in-process product is increasing based on the progress of the operation. Let C^m_i (decision variable) denote the completion time of operation O^m_i and E_k the set of operations to be performed on machine k , then the problem is as follows:

The objective function corresponds to the minimum total weighted flow time from the determined starting time to the pre-specified due date for every job. Eqs. (2) and (3) are the conjunctive and disjunctive constraints, respectively. Eq. (4) is the due-date constraint, and Eq. (5) implies that each job is available for processing at time 0.

$$\begin{aligned}
 & 1 \leq l \leq L-1 \\
 & \text{Minimize } f = \sum_{i=1}^I \sum_{l=1}^{L-1} w_{il} (C_{il}^{l+1} - p^{l+1}_i - \\
 & C_{il}^l) + w_{il}^2 (d_i - C_{il}^L) \} \quad \dots\dots (1) \\
 & \text{Subject to} \\
 & C_{mi} - C_{m-1,i} \geq p_{mi}, \quad i = 1, \dots, I, m = 2, \dots, L_i, \quad \dots\dots (2) \\
 & C_{mi} - C_{ni} \geq p_{mi} \quad \forall C_{ni} - C_{mj} \geq p_{nj}, \quad \dots\dots (3) \\
 & d_i - C_{il}^L \geq 0, \quad i = 1, \dots, I \quad \dots\dots (4) \\
 & C_{il} - p_l \geq 0, \quad i = 1, \dots, I \quad \dots\dots (5)
 \end{aligned}$$

V. PROPOSED METHODOLOGIES

A. Shuffled Frog Leaping Algorithm

In this section, an SFLA for solving the JSS problem with minimizing total holding cost and makespan criterion are proposed by population initialization, partitioning scheme, memetic evolution process, shuffling process, and a local search. SFLA was a combination of memetic Algorithm and Particle Swarm Optimization. It has been performed from the memetic evolution of a group of frogs when seeking for food. The initial population of frogs was partitioned into groups or subsets called “memeplexes” and the number of frogs in each subset was equal.

The SFLA was followed two search techniques a) local search and b) global information exchange. Based on local search to reach the makespan, the frogs in each subset improve their positions to have more foods. After a local search, obtained information based on Global information exchange between each subset was compared to other to produce best sequence way of schedule. Each operation is decided by meeting pre-specified due dates and minimizing an objective function. An initial population of the sequence generated randomly by increasing order and selected sequence divided into a number of memeplexes.

Local Search Procedure

The division is done with the high level frog (column sequence) arranged in the first memplex, the second one arranged in the second memplex, the last frog to the last memplex and repeated frog back to the next order memplex. Fitness function evaluated within the limits that the memplex is infeasible.

Global Information Exchange

The best frog memplex values were identified each subset was compared to each other to produce best sequence way of schedule. For each iteration the frogs with the best fitness and worst fitness were identified and also the frog with the makespan schedule was identified. Finally, if the convergence criteria is not satisfied the position of the worst frog for the memplex is adjusted and new subsets of memplex will be created for the next iteration.

B. SFLA Heuristics Algorithm Procedure

Start;

Step 1: Randomly generate the population size of frogs P in Feasible situation &

Initialize the population size equal to no. of memeplexes;

Step 2: For each individual population P, calculate the fitness size (i) & Calculate the size of each memplex subsets ;

Step 3: Rearrange the population size randomly;

Step4: Evaluate P based on the hierarchy order of their fitness; & Divide P into m memplexes with i=1 to no. of generations;

Step 5: Perform a Local search to Improve frog position to have the best food;

Step 6: For each memplex; Determine the best and worst frogs; Improve the worst frog position by removing worst frogs in a frame;

Step 7: Shuffle Each improved memplexes and Combine the evolved memplexes;

Step 8: Sort the population P in descending order of their fitness;

Step 9: If Convergence criteria satisfied (Make pan) move to end or else move to step 1

End;

C. Representation of Solution Seed (Sequences) in Job Shop Simulator

Consider the five-job five-machine problem as shown in Table 3. and Table 4. Suppose a seed sequence is given as [5, 4, 3, 2, 1], where 1 stands for job j1, 2 for job j2, 3 for job j3, 4 for job j4 and 5 for job j5. This sequence has to be operated five times in the same order because each job has three operations. So that the initial seed as the following format [5 4 3 2 1 5 4 3 2 1 5 4 3 2 1 5 4 3 2 1 5 4 3 2 1]. Each job has five operations, and each operation must run on all five machines with a certain time period.

Table 3: Processing Time

| Processing Time (Sec) | | | | | |
|-----------------------|----|----|----|----|----|
| MACHINE | | | | | |
| JOB | 1 | 2 | 3 | 4 | 5 |
| J1 | 64 | 7 | 74 | 54 | 80 |
| J2 | 66 | 69 | 70 | 45 | 45 |
| J3 | 31 | 68 | 60 | 98 | 10 |
| J4 | 85 | 14 | 1 | 76 | 15 |
| J5 | 44 | 18 | 90 | 13 | 91 |

Table 4: Machine Sequence

| MACHINE SEQUENCE | | | | | |
|------------------|----|----|----|----|----|
| JOB | 1 | 2 | 3 | 4 | 5 |
| J1 | m1 | m2 | m3 | m4 | m5 |
| J2 | m1 | m3 | m4 | m5 | m1 |
| J3 | m3 | m4 | m5 | m1 | m2 |
| J4 | m2 | m5 | m1 | m4 | m3 |
| J5 | m5 | m1 | m2 | m3 | m1 |

There are three 2s in the seed, which stands for the three operations of job j2. The first 2 corresponds to the first operation of job j2 which will be processed on machine 1, the second 2 corresponds to the second operation of job j2 which

will be processed on machine 3, and the third 2 corresponds to the third operation of job j2 which will be processed on machine 2. We can see that all operations for job j2 are given the same symbol 2 and then interpreted according to their orders of occurrence in the sequence of this seed. This concept is used to find the makespan for the sequences of the problems where the generated seed (job sequence) is operated equally to the number of machines represented in the particular problem.

D. Case Example

Lawrence (LA16) n number of Jobs 10 x m number of machines 10 problems is considered. Table 5 and Table 6 shows the operation sequence and its corresponding processing time [40].

Table 5: Operation Sequence Job Shop

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 6 | 9 | 8 | 7 | 2 | 0 | 4 | 3 | 5 |
| 4 | 2 | 5 | 9 | 0 | 7 | 1 | 8 | 6 | 3 |
| 3 | 2 | 8 | 1 | 4 | 9 | 7 | 6 | 0 | 5 |
| 1 | 3 | 2 | 7 | 8 | 9 | 6 | 0 | 5 | 4 |
| 2 | 0 | 5 | 6 | 7 | 1 | 4 | 9 | 3 | 8 |
| 2 | 3 | 5 | 9 | 4 | 6 | 0 | 8 | 1 | 7 |
| 3 | 2 | 0 | 1 | 9 | 8 | 6 | 5 | 4 | 7 |
| 1 | 0 | 3 | 4 | 6 | 9 | 8 | 5 | 2 | 7 |
| 4 | 2 | 8 | 5 | 3 | 7 | 1 | 6 | 9 | 0 |
| 8 | 9 | 2 | 4 | 3 | 0 | 7 | 6 | 1 | 5 |

Table 6: Processing Time of the Jobs

| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|
| 21 | 71 | 16 | 52 | 26 | 34 | 53 | 21 | 55 | 95 |
| 55 | 31 | 98 | 79 | 12 | 66 | 42 | 77 | 77 | 39 |
| 34 | 64 | 62 | 19 | 92 | 79 | 43 | 54 | 83 | 37 |
| 87 | 69 | 87 | 38 | 24 | 83 | 41 | 93 | 77 | 60 |
| 98 | 44 | 25 | 75 | 43 | 49 | 96 | 77 | 17 | 79 |
| 35 | 76 | 28 | 10 | 61 | 9 | 95 | 35 | 7 | 95 |
| 16 | 59 | 46 | 91 | 43 | 50 | 52 | 59 | 28 | 27 |
| 45 | 87 | 41 | 20 | 54 | 43 | 14 | 9 | 39 | 71 |
| 33 | 37 | 66 | 33 | 26 | 8 | 28 | 89 | 42 | 78 |
| 69 | 81 | 94 | 96 | 27 | 69 | 45 | 78 | 74 | 84 |

Table 7: Initial Operation Sequence Job Shop

| | | | | | | | | | |
|---|---|---|----|---|---|----|----|----|----|
| 1 | 1 | 8 | 6 | 2 | 3 | 9 | 7 | 4 | 5 |
| 4 | 1 | 9 | 2 | 7 | 6 | 5 | 10 | 8 | 3 |
| 3 | 7 | 1 | 2 | 9 | 1 | 8 | 4 | 6 | 5 |
| 1 | 6 | 7 | 3 | 9 | 5 | 2 | 8 | 10 | 4 |
| 2 | 4 | 6 | 10 | 1 | 3 | 5 | 7 | 9 | 8 |
| 2 | 1 | 9 | 3 | 6 | 1 | 5 | 4 | 8 | 7 |
| 3 | 6 | 1 | 2 | 8 | 4 | 10 | 9 | 5 | 7 |
| 1 | 8 | 4 | 10 | 9 | 2 | 3 | 6 | 5 | 7 |
| 4 | 1 | 5 | 2 | 7 | 9 | 8 | 3 | 6 | 10 |
| 8 | 7 | 4 | 9 | 1 | 1 | 2 | 3 | 6 | 5 |

Table 8: Initial Processing Time of the Jobs

| | | | | | | | | | |
|----|----|----|----|----|----|---|---|----|----|
| 21 | 53 | 52 | 71 | 34 | 55 | 1 | 2 | 21 | 95 |
| 55 | 42 | 79 | 31 | 66 | 77 | 9 | 1 | 77 | 39 |
| 34 | 43 | 19 | 64 | 79 | 83 | 6 | 9 | 54 | 37 |
| 87 | 41 | 38 | 69 | 83 | 77 | 8 | 2 | 93 | 60 |
| 98 | 96 | 75 | 44 | 49 | 17 | 2 | 4 | 77 | 79 |
| 35 | 95 | 10 | 76 | 9 | 7 | 2 | 6 | 35 | 95 |
| 16 | 52 | 91 | 59 | 50 | 28 | 4 | 4 | 59 | 27 |
| 45 | 14 | 20 | 87 | 43 | 39 | 4 | 5 | 9 | 71 |
| 33 | 28 | 33 | 37 | 8 | 42 | 6 | 2 | 89 | 78 |
| 69 | 45 | 96 | 81 | 69 | 74 | 9 | 2 | 78 | 84 |

Initiations

Initial population of sequence generated randomly by increasing order and selected sequence divided into number of memplexes are shown in Table 7 & Table 8.

Local Search

The division is done with the high -level frog (column sequence) arranged in the first memplex, the second one arranged in the second memplex, the last frog to the last memplex and repeated frog back to the next order memplex are shown in Table 9. Fitness function evaluated within the limits that the memplex are infeasible.

Table 9 Column Sequence Arrangement

| | | | | | | | | |
|---|---|---|---|----|----|----|---|---|
| 8 | 6 | 9 | 4 | 10 | 5 | 1 | 2 | 3 |
| 9 | 2 | 4 | 1 | 6 | 5 | 2 | 3 | 8 |
| 1 | 7 | 5 | 8 | 4 | 1 | 10 | 6 | 3 |
| 1 | 5 | 7 | 3 | 10 | 8 | 2 | 6 | 4 |
| 3 | 7 | 9 | 8 | 10 | 1 | 5 | 4 | 6 |
| 4 | 3 | 7 | 9 | 5 | 8 | 10 | 6 | 2 |
| 1 | 5 | 1 | 9 | 7 | 4 | 2 | 3 | 8 |
| 2 | 5 | 3 | 1 | 6 | 8 | 10 | 7 | 4 |
| 6 | 8 | 4 | 2 | 9 | 10 | 7 | 5 | 1 |
| 8 | 6 | 9 | 4 | 10 | 5 | 1 | 3 | 2 |

Global Information Exchange

The best frog memplex values were identified each subset was compared to each other to produce best sequence way of schedule.

Iterations

For each iteration process, the frogs with the best fitness and worst fitness were identified and also the frog with the makespan schedule was identified. Finally, if the convergence criteria are not satisfied the position of the worst frog for the memplex is adjusted and new subsets of memplex will be created for the next iteration. This procedure is repeated for the desired number of iterations to reach the optimal result.

VI. NUMERICAL ILLUSTRATION

Stage 1: Initiations

The initial population of job seed sequences is generated randomly by increasing order and selected sequence divided into the number of memplexes.

Sequence No. 1

[(2-1-8-10-6-3-9-7-4-5)(7-4-9-1-2-6-5-10-8-3)(10-3-1-9-2-7-8-4-6-5)(1-2-6-4-10-3-5-7-9-8)(6-10-9-2-3-1-5-4-8-7)(9-1-7-6-3-5-2-8-10-4)(8-3-1-6-2-4-10-9-5-7)(4-9-10-8-1-2-3-6-5-7)(7-4-5-1-2-9-8-3-6-10)(2-1-8-10-6-3-9-7-4-5)]

THC (with Tardy Job) = 167345.78

Makespan = 946

THC (with no Tardy Job) = 159875.543

Sequence No. 2

[(9-1-7-6-3-5-2-8-10-4)(7-4-9-1-2-6-5-10-8-3)(10-3-1-9-2-7-8-4-6-5)(1-2-6-4-10-3-5-7-9-8)(6-10-9-2-3-1-5-4-8-7)(2-1-8-10-6-3-9-7-4-5)(8-3-1-6-2-4-10-9-5-7)(4-9-10-8-1-2-3-6-5-7)(7-4-5-1-2-9-8-3-6-10)(2-1-8-10-6-3-9-7-4-5)]

THC (with Tardy Job) = 167316.456

Makespan = 945

THC (with no Tardy Job) = 159645.563

Sequence No. 3

[(7-4-9-1-2-6-5-10-8-3)(2-1-8-10-6-3-9-7-4-5)(10-3-1-9-2-7-8-4-6-5)(1-2-6-4-10-3-5-7-9-8)(2-1-8-10-6-3-9-7-4-5)(6-10-9-2-3-1-5-4-8-7)(9-1-7-6-3-5-2-8-10-4)(9-1-7-6-3-5-2-8-10-4)(8-3-1-6-2-4-10-9-5-7)(7-4-5-2-9-8-3-6-10)(4-9-10-8-1-2-3-6-5-7)]

THC (with Tardy Job) = 167254.663

Makespan = 943

THC (with no Tardy Job) = 159573.423

Sequence No. 4

[(1-4-9-7-2-6-5-10-8-3)(10-1-8-2-6-3-9-7-4-5)(9-3-1-10-2-7-8-4-6-5)(4-2-6-1-10-3-5-7-9-8)(8-1-2-10-6-3-9-7-4-5)(6-10-9-2-3-1-5-4-8-7)(9-1-7-6-3-5-2-8-10-4)(9-1-7-6-3-5-2-8-10-4)(8-3-1-6-2-4-10-9-5-7)(7-4-5-2-9-8-3-6-10)(4-9-10-8-1-2-3-6-5-7)]

THC (with Tardy Job) = 167232.68

Makespan = 944

THC (with no Tardy Job) = 159426.75

Sequence No. 5

q[(7-2-3-9-7-10-4-5-6-8)(8-10-9-2-7-3-6-5-4-1)(2-9-5-8-7-6-10-1-3-4)(1-3-5-4-10-6-2-7-9-8)(3-5-10-9-7-1-4-2-8-6)(3-5-10-9-7-1-4-2-8-6)(8-3-4-2-7-10-6-5-9-1)(9-2-3-7-6-10-1-8-4-5)(3-5-4-10-6-2-7-9-8-1)(5-10-9-7-1-4-2-3-8-6)]

THC (with Tardy Job) = 168765.42

Makespan = 942

THC (with no Tardy Job) = 156456.75

Sequence No. 6

[(10-5-3-9-7-1-6-2-8-4)(7-3-4-2-8-10-5-6-9-1)(2-9-5-3-7-6-10-4-8-1)(1-3-5-6-10-4-2-7-9-8)(9-5-10-3-7-1-4-2-8-6)]

6)(3-5-4-9-7-1-10-2-8-6)(8-3-4-2-7-10-5-6-9-1)(3-2-9-7-6-10-1-8-4-5)(3-5-4-10-6-2-7-9-8-1)(5-10-9-7-1-4-2-3-8-6)]

THC (with Tardy Job) = 165235.42 Makespan = 941

THC (with no Tardy Job) = 156224.21

Sequence No. 7

[(4-9-10-8-1-2-3-6-5-7)(10-1-8-2-6-3-9-7-4-5)(9-3-1-10-2-7-8-4-6-5)(4-2-6-1-10-3-5-7-9-8)(8-1-2-10-6-3-9-7-4-5)(6-10-9-2-3-1-5-4-8-7)(9-1-7-6-3-5-2-8-10-4)(9-1-7-6-3-5-2-8-10-4)(8-3-1-6-2-4-10-9-5-7)(7-4-5-2-9-8-3-6-10)(8-3-4-2-7-10-5-6-9-1)]

THC (with Tardy Job) = 166522.64 Makespan = 945

THC (with no Tardy Job) = 157523.36

Sequence No. 8

[(9-10-1-7-2-4-3-6-5)(8-5-9-4-6-1-3-2-10-7)(2-8-9-3-7-5-4-6-10-1)(1-3-5-6-10-4-2-7-9-8)(9-5-10-3-7-1-4-2-8-6)(3-5-4-9-7-1-10-2-8-6)(8-3-4-2-7-10-5-6-9-1)(3-2-9-7-6-10-1-8-4-5)(8-3-4-2-7-10-6-5-9-1)(9-2-3-7-6-10-1-8-4-5)]

THC (with Tardy Job) = 165823.35 Makespan = 941

THC (with no Tardy Job) = 154424.99

Sequence No. 9

[(9-7-4-3-1-10-5-6-8-5)(2-5-6-8-9-2-6-7-4-1)(7-5-10-1-2-6-9-8-3-4)(9-2-1-6-8-3-7-4-5-10)(5-6-4-8-10-7-3-9-2-1)(8-4-6-9-3-1-10-2-7-5)(5-1-8-6-9-2-10-7-3-4)(8-4-6-9-3-1-10-2-7-5)(3-5-4-9-7-1-10-2-8-6)]

THC (with Tardy Job) = 164667.35 Makespan = 940

THC (with no Tardy Job) = 154298.99

Sequence No. 10

[(2-4-6-1-3-5-9-10-8-7)(7-5-6-9-10-4-5-2-3-1)(4-5-7-9-10-1-2-4-3-5)(3-2-9-7-6-10-1-8-4-5)(9-1-7-6-3-5-2-8-10-4)(2-1-5-3-7-10-8-9-6)(2-8-9-3-7-4-5-1-10-6)(10-8-9-7-3-5-4-6-10-2)(4-2-6-1-10-3-8-7-9-5)(7-1-2-10-6-3-9-8-4-5)]

THC (with Tardy Job) = 165543.35 Makespan = 941

THC (with no Tardy Job) = 154875.76

Stage 2: Population Creation

For each individual population i in P calculate the fitness function $f(i)$. Based on the fitness function calculate the size of each memplex subsets and also randomly generate the population of the job sequence. The fitness function is 5, their sequences memplexes are (2, 5) & (6, 10). The next step of operation sequences are grouped randomly.

Set 1

Set 2

Sequence No. 2

Sequence No. 1

Sequence No. 5

Sequence No. 7

| | |
|-----------------|----------------|
| Sequence No. 6 | Sequence No. 3 |
| Sequence No. 8 | Sequence No. 5 |
| Sequence No. 10 | Sequence No. 4 |

Stage 3: Mutation

In the mutation operation, memplex subsets are generated using the mutation strategy to find the population P in descending order based on their fitness. Then evaluate and Divide the population sequence P into m memplexes with consideration of populations which is selected randomly. Two random positions are chosen and population mutation is performed in between two positions in set 2. 5th and 7th sequences positions are chosen randomly and inverse mutation is performed. Their next sequence orders are (1,5,7,3,4)& (2,8,5,6,10)

Table 10: Mutant Operation Sequence

| | | | | | | | | | | |
|---|----|---|---|----|----|----|----|---|---|---|
| 8 | 7 | 6 | 9 | 4 | 10 | 5 | 1 | 3 | 2 | Make span: 938 THC (with Tardy Jobs): 160248.14 THC(with Tardy Jobs): 158722.76 |
| 2 | 9 | 7 | 6 | 2 | 5 | 3 | 1 | 4 | 8 | |
| 9 | 10 | 7 | 6 | 10 | 1 | 4 | 8 | 5 | 3 | |
| 9 | 1 | 5 | 7 | 3 | 10 | 8 | 2 | 6 | 4 | |
| 2 | 3 | 7 | 9 | 8 | 10 | 1 | 5 | 4 | 6 | |
| 1 | 4 | 3 | 7 | 9 | 5 | 8 | 10 | 6 | 2 | |
| 6 | 10 | 5 | 1 | 9 | 7 | 4 | 2 | 3 | 8 | |
| 9 | 2 | 5 | 3 | 1 | 6 | 8 | 10 | 7 | 4 | |
| 3 | 6 | 8 | 4 | 2 | 9 | 10 | 7 | 5 | 1 | |
| 7 | 8 | 6 | 9 | 4 | 10 | 5 | 1 | 3 | 2 | |

Stage 4: Crossover

Mutation operation generators are used to generate a trial function vector. In this operation, a random population sequence is generated in between 0 to 1 and if the random number is less than the crossover constant value copy the target value otherwise the mutant operation sequence value will be changed as 0 or 1 for i=1 to the number of generations are shown in Table.10.

Stage 5: Local Search

The division is done with the high- level frog (column sequence) arranged in the first memplex, the second one arranged in the second memplex, the last frog to the last memplex and repeated frog back to the next order memplex. Fitness function evaluated within the limits that the memplex are infeasible. Then Perform the Local search to improve the frog position to have the best food.

Stage 6: Global Information Exchange

The best frog memplex values were identified with each subset was compared to each other to produce best sequence way of schedule. For each memplex, determine the best and worst frogs and improve the worst frog position by removing worst frogs in the operation sequence frame.

Stage 7: Shuffling

The trial sequence obtained by the crossover operation generation is compared with the target sequence to

determine the jobs and machine schedule that participates in the next generation and the fittest is passed on to the next generation. If the objective value $f(i)$ is lower than required processing value P_i), then random value replaces the best compared value, Otherwise, consider the best fitness. Finally, Shuffle each improved memplexes and Combine the evolve memplexes and also Sort the population P in descending order of their best fitness value.

Stage 7: Iterations

For each iteration, the frogs with the best fitness and worst fitness were identified and also the frog with the makespan schedule was identified.

Stage 8: Control Parameters

Finally, if the convergence criteria are not satisfied the position of the worst frog for the memplex is adjusted and new subsets of memplex will be created for the next iteration. This procedure is repeated for the desired number of iterations to reach the optimal result.

A. Final Result obtained using SFLA Algorithm

The best solutions found in 100 iterations of the local search process and Global information exchange for minimizing total holding cost with tardy jobs and without consideration of tardy jobs are listed in Table 11.

Case (i) Consideration of all production tasks with the Same Function

Case (ii) Consideration of all production tasks with the individual functions results are shown in Table 12.

Table 11: Results Obtained after 100 Iterations

| SFLA | Local Search | | | Glopal Information Exchange | | |
|------------------|--------------|------------------------------------|---------------------------------------|-----------------------------|------------------------------------|---------------------------------------|
| Iteration's | Makespan | Total Holding Cost with Tardy Jobs | Total Holding Cost with no Tardy Jobs | Makespan | Total Holding Cost with Tardy Jobs | Total Holding Cost with no Tardy Jobs |
| First Iteration | 944 | 162557.0299 | 159798.6 | 944 | 161357.091 | 151798.6 |
| Second Iteration | 942 | 161890.5434 | 158548.6 | 945 | 161093.534 | 152473.6 |
| Third Iteration | 940 | 161224.0569 | 156595.6 | 943 | 160829.977 | 151346.7 |
| Last Iteration | 940 | 159557.5704 | 154665.6 | 939 | 158192.673 | 150268.6 |

Table 12: Individual Function Considerations

| Individual Function Considerations | | | | | | | | |
|------------------------------------|----------|---------------------|---------------------|-------------------------------|------------------------|-------------|----------------|--------------|
| Description | Makespan | Mean Tardiness Cost | Mean Earliness Cost | Finished Product Holding Cost | Inprocess Holding Cost | Only THC | THC with Tardi | THC with all |
| Notations | | $\sum ti$ | $\sum Ei$ | $\sum Pi$ | $\sum Ii$ | $\sum Hij$ | $\sum hiti$ | c(s) |
| Sequence No. 1 | 947 | 121.89 | 462.534 | 64550 | 97790.06 | 162340.06 | 162461.95 | 161999.416 |
| Sequence No. 2 | 946 | 131.3 | 642.643 | 63299 | 98772.4 | 162071.4 | 162202.7 | 161560.057 |
| Sequence No. 3 | 945 | 131.4 | 432.335 | 63433 | 98676.13 | 162109.13 | 162240.53 | 161808.195 |
| Sequence No. 4 | 944 | 141.7 | 462.305 | 62573.6 | 99298.93 | 161872.53 | 162014.23 | 161551.925 |
| Sequence No. 5 | 943 | 141.95 | 447.2055 | 61975.2 | 99741.96 | 161717.16 | 161859.11 | 161411.9045 |
| Sequence No. 6 | 942 | 162.2 | 426.106 | 61736.7 | 100185 | 161921.7 | 162083.9 | 161657.794 |
| Sequence No. 7 | 941 | 162.7 | 405.0065 | 60738.1 | 100628.038 | 161366.138 | 161528.838 | 161123.8315 |
| Sequence No. 8 | 940 | 169.7 | 383.907 | 60169.3 | 101071.07 | 161240.37 | 161410.07 | 161026.163 |
| Sequence No. 9 | 938 | 176.71 | 362.8075 | 59671.4 | 101514.1083 | 161185.5083 | 161362.2183 | 160999.4108 |
| Sequence No. 10 | 936 | 183.6 | 341.708 | 59032.6 | 101957.14 | 160989.74 | 161173.34 | 160831.632 |

B. Practical Applications of Proposed SFL Algorithm for Minimizing THC with and Without Tardy Jobs

Table 13: Customer Order

| Date of Order | Product Required | Needed Quantity | Due Date (week) |
|---------------|------------------|-----------------|-----------------|
| 25/03/2012 | Finned Tubes | 1000 | 1 |
| | | 1500 | 3 |
| | | 1400 | 4 |
| | | 2000 | 5 |
| | | 1400 | 6 |
| | | 3000 | 8 |

The proposed SFL algorithm can be successfully implemented in industries handling a wide variety of products in small volumes and the industries working with general purpose machines which can handle different operations. The job processing and waiting times can be conveniently cut down, machine loads can be balanced and also the user has a choice of choosing a solution from the set of alternative solutions as per his desired objective criteria.

Customer Order

Table.13 and Figure.1 shows the customer order for manufacturing finned tubes. It gives details of needed quantity in terms of the week.

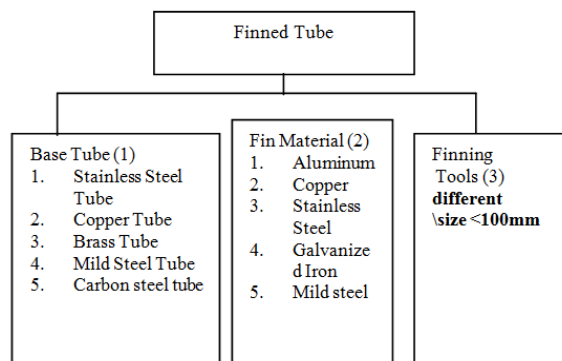


Figure 1: Product Structure of Finned Tubes

Inventory Record Details

The Inventory record details for base Tube (1) is shown in Table 14 and for Fin, material is shown in Table 15. The final results obtained after 100 iterations for the case study problem is listed in Table 16. Minimizing the total holding cost, Meeting due dates, Minimizing tardiness, Earliness are the most important goal of scheduling to avoid the delay penalties including customer's bad impression, lost future sales, etc.. Many research papers have focused on due-date oriented functions, whereas the main aim of optimizing the makespan is to minimizing costs and maximizing the output. In Many research articles and studies, Heuristics Algorithms are competent and proves to be a good problem-solving technique for job shop scheduling.

Table 14: Inventory Record Details for Base Tube (1)

| Job/Part No | Part Description (Fin Strip) | Lead Time (week) | Stock on Hand(size) |
|----------------|----------------------------------|------------------|-------------------------------|
| j ₁ | Stainless Steel | 3 weeks | Min 6mm to Max 15mm in 150 kg |
| j ₂ | Copper Tube | 4 weeks | Min 6mm to Max 15mm in 200 kg |
| j ₃ | Brass Tube | 5 weeks | Min 6mm to Max 15mm in 150 kg |
| j ₄ | Mild Steel Tube, Galvanized Iron | 2 Weeks | Min 6mm to Max 15mm in 100 kg |
| j ₅ | Aluminum, Copper, Mild Steel | 3 Weeks | Min 6mm to Max 15mm in 350 kg |

Table 15: Inventory Record Details for Fin Material (2)

| Job/ Part No | Part Description | Lead Time (week) | Stock on hand |
|----------------|--------------------------|------------------|----------------------------------|
| j ₁ | Stainless Steel Tube | 3 weeks | 3m in 200 nos 2.5m in 50 nos |
| j ₂ | Copper Tube Aluminium | 4 weeks | 3 m in 100 nos 6 m in 150 nos |
| j ₃ | Copper Tube Aluminium | 5 weeks | 3 m in 50 nos 6 m in 100 nos |
| j ₄ | Aluminium | 2 Weeks | 6 m in 50 nos |
| j ₅ | Carbon Steel Tube | 3 Weeks | 6m in 350nos |

C. Benchmark Problem Solutions

Five instances of size ($n \times m = 10 \times 10$) denoted as (LA16-LA20) from Lawrence [56] with different 't' parameter for to control the due dates. Setting $t = 2.5, 3.5$ where the due-date constraint is loose and $t = 1.8, 1.9$ where the due-date constraint is strict.

Benchmark problem which contains Ten jobs and ten machines for instances LA 16- LA 20 taken from Lawrence was tested with the proposed SFL algorithm and the results are compared with Sheep Frog Leaping Algorithm, Artificial immune system and Heuristic Shifting Bottleneck (HSB) procedure reported in the literature (Reference). These instances were tested for minimum makespan problem, Total holding cost problem and due date consideration problem. The best

objective function value solution, computing time is investigated for single objective function and separate objective functions. The results are shown in Table 17 and Table 18. Furthermore, in order to investigate the performance of the proposed heuristic approaches for more problems, the LA36-LA40 of 15 jobs and 15 machines benchmark problems are also considered. The literature heuristics procedures are needs the enormous computation time to solve the 15 jobs and 15 machines problems. The proposed SFL algorithm has achieved good solutions for the 15 jobs and 15 machines problems with lesser computing time.

Table 16: Final Results Obtained for the Case Study Problem

| Description | Individual Function Considerations | | | | | | | |
|----------------|------------------------------------|---------------------|---------------------|-------------------------------|------------------------|------------|---------------|--------------|
| | Makespan | Mean Tardiness Cost | Mean Earliness Cost | Finished Product Holding Cost | Inprocess Holding Cost | Only THC | THC with Tard | THC with all |
| Notations | | $\sum t_i$ | $\sum E_i$ | $\sum P_i$ | $\sum I_i$ | $\sum H_j$ | $\sum H_{it}$ | e(s) |
| Sequence No. 2 | 188.12 | 11.09 | 2.106 | 59356.6 | 61842.6816 | 121199.282 | 121210.372 | 121208.2656 |
| Sequence No. 3 | 188.06 | 10.99 | 2.0065 | 58820.5 | 61246.6433 | 120067.173 | 120078.163 | 120076.1568 |
| Sequence No. 4 | 184.6 | 10.59 | 1.907 | 58704.46 | 61208.605 | 119913.065 | 119923.655 | 119921.748 |
| Sequence No. 5 | 183.785 | 10.265 | 1.8075 | 57268.4 | 60470.5667 | 117738.957 | 117749.222 | 117747.4142 |
| Sequence No. 6 | 182.252 | 10.18 | 1.708 | 56652.32 | 60232.5284 | 116884.848 | 116895.028 | 116893.3204 |
| Sequence No. 7 | 180.719 | 10.095 | 1.6085 | 55376.3 | 59294.4901 | 114670.74 | 114680.835 | 114679.2266 |
| Sequence No. 8 | 179.186 | 10.01 | 1.509 | 55070.18 | 59516.4518 | 114586.632 | 114596.642 | 114595.1328 |
| Sequence No. 9 | 177.653 | 9.925 | 1.4095 | 54224.1 | 59118.4135 | 113342.524 | 113352.449 | 113351.039 |
| Sequence No. 1 | 176.12 | 9.84 | 1.31 | 53648.04 | 58580.3752 | 112228.415 | 112238.255 | 112236.9452 |

Table 17: Ten Jobs and Ten Machines for Instances LA 16- LA 20

| Heuristics Algorithm | Problem | t = 1.8 | | t = 1.9 | | t = 2.5 | | t = 3.5 | |
|----------------------|---------|---------------|-----------------|---------------|-----------------|---------------|-----------------|---------------|-----------------|
| | | Best Value | CPU time in sec | Best Value | CPU time in sec | Best Value | CPU time in sec | Best Value | CPU time in sec |
| SFHM Algorithm | LA 16 | 155458 | 14 | 162765 | 11.5 | 181715 | 19.6 | 219945 | 11.5 |
| | LA 17 | 140071 | 2.8 | 140902 | 10.8 | 164089 | 4.5 | 201687 | 19.8 |
| | LA 18 | 141624 | 21.5 | 139986 | 18.6 | 159937 | 17.2 | 189851 | 17.3 |
| | LA 19 | 112760 | 30.8 | 111971 | 21 | 120963 | 28.6 | 133792 | 23.7 |
| | LA 20 | 113499 | 9.6 | 170635 | 14.5 | 195046 | 26.5 | 244373 | 21.9 |
| AIS Algorithm | LA 16 | 160112 | 25 | 164354 | 18 | 184679 | 29.5 | 220567 | 15.7 |
| | LA 17 | 144894 | 3 | 146106 | 19.4 | 166005 | 6 | 202075 | 32.3 |
| | LA 18 | 143004 | 29.7 | 143348 | 30.9 | 160568 | 21.4 | 190933 | 21.4 |
| | LA 19 | 114057 | 38.5 | 112977 | 34.5 | 122901 | 44.8 | 134027 | 44 |
| | LA 20 | 115973 | 12.1 | 171012 | 20.7 | 197634 | 40.2 | 245416 | 38.6 |
| HSB Algorithm | LA 16 | 160374 | 28.2 | 165402 | 25.2 | 186995 | 32.7 | 221604 | 19.1 |
| | LA 17 | 146206 | 3.2 | 147518 | 25.2 | 167567 | 6.3 | 202933 | 38.2 |
| | LA 18 | 143833 | 33.4 | 145730 | 35.1 | 162154 | 22.9 | 191316 | 22.2 |
| | LA 19 | 114242 | 43.7 | 115251 | 49 | 123436 | 56.7 | 135171 | 57.8 |
| | LA 20 | 116339 | 13.8 | 171343 | 25.5 | 199713 | 42.6 | 247715 | 49.3 |
| SFL Algorithm | LA 16 | 155458 | 14 | 161358 | 10.8 | 173433 | 18.4 | 214556 | 11.1 |
| | LA 17 | 139871 | 3.9 | 140589 | 10.5 | 163662 | 4.3 | 201124 | 18.6 |
| | LA 18 | 124284 | 21.1 | 135417 | 17.8 | 158252 | 16.8 | 187422 | 16.3 |
| | LA 19 | 111697 | 29.3 | 111738 | 20.9 | 120534 | 27.5 | 132597 | 226 |
| | LA 20 | 112339 | 9.5 | 168905 | 13.2 | 194872 | 25.2 | 241212 | 20.3 |

Table 18: Final Result comparisons of SFLA with other Heuristics

| Results Comparison of SFLA with SFHM, AIS and HSB | | | | | | | | | | | | | |
|---|----------|------------------------|------|------|----|----------------------------|------|----|---------------------------|------|----|---------------------------|-------------------------------|
| SJ No | Instance | Problem Size (Ji x Mi) | SFLA | SFHM | IP | % of Improvement over SFHM | AIS | IP | % of Improvement over AIS | HSB | IP | % of Improvement over HSB | Overall % of Improvement (IP) |
| 1 | LA 16 | 10x10 | 939 | 945 | 6 | 5.96 | 946 | 7 | 6.95 | 948 | 9 | 8.91 | 7.27 |
| 2 | LA 17 | 10x10 | 783 | 784 | 1 | 1.00 | 791 | 8 | 7.92 | 789 | 6 | 5.95 | 4.96 |
| 3 | LA 18 | 10x10 | 846 | 848 | 2 | 2.00 | 847 | 1 | 1.00 | 849 | 3 | 2.99 | 1.99 |
| 4 | LA 19 | 10x10 | 841 | 842 | 1 | 1.00 | 851 | 10 | 9.88 | 853 | 12 | 11.83 | 7.57 |
| 5 | LA 20 | 10x10 | 898 | 902 | 4 | 3.98 | 899 | 1 | 1.00 | 901 | 3 | 2.99 | 2.66 |
| 6 | LA 36 | 15x15 | 1266 | 1268 | 2 | 2.00 | 1268 | 2 | 2.00 | 1273 | 7 | 6.96 | 3.65 |
| 7 | LA 37 | 15x15 | 1389 | 1401 | 12 | 11.90 | 1405 | 16 | 15.82 | 1397 | 8 | 7.95 | 11.89 |
| 8 | LA 38 | 15x15 | 1196 | 1196 | 0 | 0.00 | 1196 | 0 | 0.00 | 1196 | 0 | 0.00 | 0.00 |
| 9 | LA 39 | 15x15 | 1233 | 1233 | 0 | 0.00 | 1234 | 1 | 1.00 | 1233 | 0 | 0.00 | 0.33 |
| 10 | LA 40 | 15x15 | 1223 | 1224 | 1 | 1.00 | 1227 | 4 | 3.99 | 1238 | 15 | 14.82 | 6.60 |

VII. CONCLUSIONS

To avoid customer's bad impression and To improve the customer's Satisfaction by delivering the jobs within the due date is a very important criterion in the manufacturing system. In order to avoid delay penalties including customer's bad impression, cost of lost future sales and rush shipping cost, due date constraints are considered. The objective considered in this paper is minimizing total holding cost which means a sum of product inventory cost and in process inventory cost with consideration of tardy jobs and without consideration of tardy jobs. And also a heuristic Shuffled frog leaping algorithm is proposed for the job-shop scheduling problem to minimize total holding cost and make the pan. Strict due date parameter and lose due date parameter is used for analyzing the total holding cost. The proposed heuristics are used for testing Lawrence 10 x 10 and 15 x 15 benchmark problems. Results show that SFL algorithm produces good quality results compared with other Heuristics approach procedures.

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